FORMULÁRIO

- **Conservação de Massa:** $\frac{\partial}{\partial t}\int_{V.C} \rho\, d\forall + \int_{SC} \rho\, \vec{V}\bullet\vec{n}\, dA = 0$

$$\frac{\partial \rho}{\partial t} + \text{div}(\rho\,\vec{V}) = 0 \quad \textbf{ou} \quad \frac{D\rho}{Dt} + \rho\,\vec{\nabla}\bullet\vec{V} = 0$$

  - derivada material, ou total ou substantiva $\frac{D(\;)}{Dt} = \frac{\partial(\;)}{\partial t} + \vec{V}\bullet\vec{\nabla}(\;)$

Obs: $\vec{V} = u\,\vec{i} + v\,\vec{j} + w\,\vec{k}$, $\vec{V} = u_r\,\vec{e}_r + u_\theta\,\vec{e}_\theta + u_z\,\vec{e}_z$

1. Coordenadas cartesianas: $\Rightarrow$ $\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} + \frac{\partial \rho w}{\partial z} = 0$
2. Coordenadas cilíndricas: $\Rightarrow$ $\frac{\partial \rho}{\partial t} + \frac{\partial r \rho u_r}{r\,\partial r} + \frac{\partial \rho v_\theta}{r\,\partial \theta} + \frac{\partial \rho v_z}{\partial z} = 0$

- **função de corrente** ψ $\Rightarrow$ $u = \frac{\partial \psi}{\partial y}$, $v = -\frac{\partial \psi}{\partial x}$ ou $u_r = \frac{\partial \psi}{r\,\partial \theta}$, $u_\theta = -\frac{\partial \psi}{\partial r}$

- **função potencial** ϕ $\Rightarrow u = -\frac{\partial \phi}{\partial x}, \quad v = -\frac{\partial \phi}{\partial y}, \quad w = -\frac{\partial \phi}{\partial z}$ ou

$$u_r = -\frac{\partial \phi}{\partial r}, \quad u_\theta = -\frac{1}{r}\frac{\partial \phi}{\partial \theta}, \quad u_z = -\frac{\partial \phi}{\partial z}$$

- **Rotação de Fluidos** $\Rightarrow$ $\vec{\omega} = \omega_x\,\vec{i} + \omega_y\,\vec{j} + \omega_z\,\vec{k}$, $\vec{\omega} = \frac{1}{2}\nabla\times\vec{V}$

$$\omega_x = \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right), \quad \omega_y = \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right), \quad \omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

- **Equação de Conservação de Quantidade de Movimento Linear (2ª Lei de Newton)**

$$\sum\vec{F}_s + \sum\vec{F}_c = \frac{\partial}{\partial t}\int_{V.C}\vec{V}\,\rho\, d\forall + \int_{SC}\vec{V}\;\rho\,\vec{V}\bullet\vec{n}\, dA$$

$$\rho\frac{D\vec{V}}{Dt} = \rho\,\vec{g} - \textbf{grad}\,p + \textbf{div}[\underline{\underline{\tau}}]$$

- **Equação de Navier-Stokes**

$$\rho\frac{D\vec{V}}{Dt} = \rho\vec{g} - \nabla p + \mu\nabla^2\vec{V}$$

Equação de Euler: $\rho\frac{D\vec{V}}{Dt} = \rho\,\vec{g} - \textbf{grad}\,p$

- **coordenadas cartesianas: :** $\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho\,g_x - \frac{\partial p}{\partial x}$,

$$\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho\,g_y - \frac{\partial p}{\partial y}, \quad \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho\,g_z - \frac{\partial p}{\partial z}$$

- **Equação de Bernoulli:** $\int\frac{\partial V_s}{\partial t}ds + \frac{V_s^2}{2} + g\,z + \int\frac{dp}{\rho} = \text{constante}$

- **Equação de Conservação de Energia(1ª Lei da termodinâmica) [$\dot{Q}$ entrando positivo, $\dot{W}$ saindo positivo]**

$$\dot{Q} - \dot{W}_\mu - \dot{W}_e - \dot{W}_{outros} = \frac{\partial}{\partial t}\int_{V.C} e\,\rho\, d\forall + \int_{SC}\left[e + \frac{p}{\rho}\right]\rho\,\vec{V}\bullet\vec{n}dA \qquad e = i + \frac{V^2}{2} + g\,z$$

- **Equação de Bernoulli Modificada:** $-\frac{\dot{W}_e}{\dot{m}\,g} = \left[\frac{p_2}{\rho\,g} + \frac{V_2^2}{2\,g} + z_2\right] - \left[\frac{p_1}{\rho\,g} + \frac{V_1^2}{2\,g} + z_1\right] + h_{L1,2}$

- **Perda de Carga:** $h_{L12} = f\,\frac{L}{D}\,\frac{V_m^2}{2\,g}$ **fator de atrito:** $f = \frac{\left(-\frac{dp}{dx}\right)D_h}{\frac{1}{2}\rho\,V_m^2} = \frac{4\,\tau_s}{\frac{1}{2}\rho\,V_m^2}$

- **Nº. de Reynolds**: $Re = \frac{\rho V_m D_h}{\mu}$ **Diâmetro Hidráulico:** $D_h = \frac{4\,A_t}{P_m}$

- **Potência:** $Pot = F\,V$, $Pot = Q\,\Delta p$, $Q = V_m A_T = \int_A V\,dA$

- **Equação de Navier-Stokes: coordenadas cartesianas:**

direção x: $\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}\right) = \rho\,g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$

direção y: $\rho\left(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}\right) = \rho\,g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$

direção z: $\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right) = \rho\,g_z - \frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$

- **Eq de Navier-Stokes coordenadas cilíndricas**

direção r:
$$\rho\left[\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + u_\theta\frac{\partial u_r}{r\,\partial\theta} + u_z\frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r}\right] = \rho\,g_r - \frac{\partial p}{\partial r} +$$
$$\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_r}{\partial r}\right) - \frac{u_r}{r^2} + \frac{\partial^2 u_r}{r^2\,\partial\theta^2} + \frac{\partial^2 u_r}{\partial z^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial\theta}\right]$$

direção θ
$$\rho\left[\frac{\partial u_\theta}{\partial t} + u_r\frac{\partial u_\theta}{\partial r} + u_\theta\frac{\partial u_\theta}{r\,\partial\theta} + u_z\frac{\partial u_\theta}{\partial z} + \frac{u_r\,u_\theta}{r}\right] = \rho\,g_\theta - \frac{\partial p}{r\,\partial\theta} +$$
$$\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_\theta}{\partial r}\right) - \frac{u_\theta}{r^2} + \frac{\partial^2 u_\theta}{r^2\,\partial\theta^2} + \frac{\partial^2 u_\theta}{\partial z^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial\theta}\right]$$

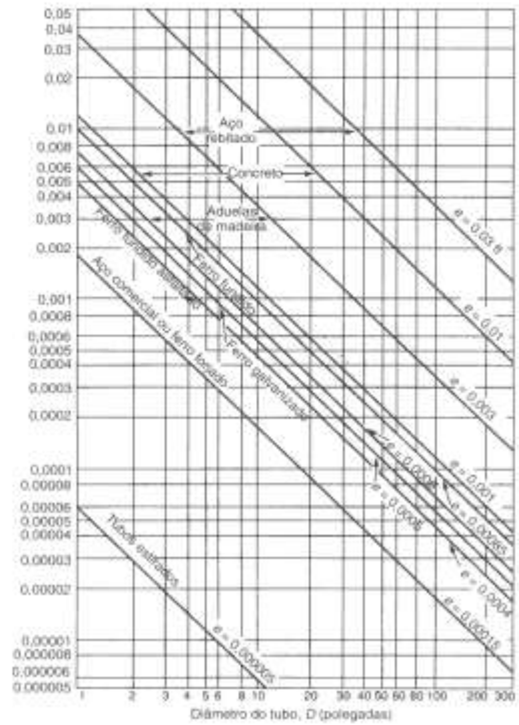
direção z: $\rho\left[\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + u_\theta\frac{\partial u_z}{r\,\partial\theta} + u_z\frac{\partial u_z}{\partial z}\right] = \rho\,g_z - \frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_z}{\partial r}\right) + \frac{\partial^2 u_z}{r^2\,\partial\theta^2} + \frac{\partial^2 u_z}{\partial z^2}\right]$

- **conversões:** 1 in = 25,4 mm 1HP = 745 Watts K= C + 273 R = F + 460

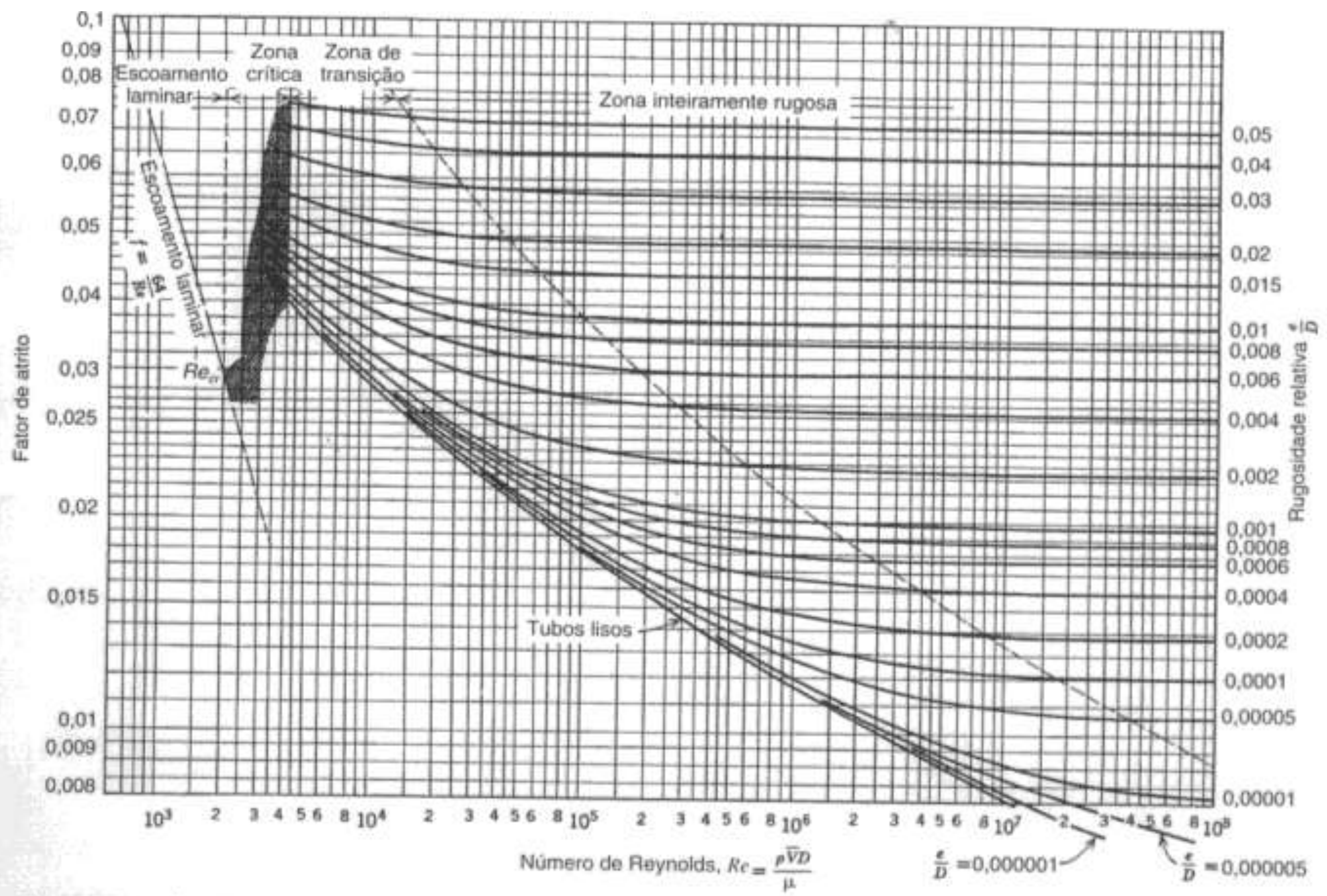
Integrais: $\int sen\theta\, d\theta = -\cos\theta$, $\int\cos\theta\, d\theta = sen\theta$; $\int sen^2\theta\, d\theta = \frac{\theta}{2} - \frac{sen(2\theta)}{4}$, $\int\cos^2\theta\, d\theta = \frac{\theta}{2} + \frac{sen(2\theta)}{4}$

$\int sen^3\theta\, d\theta = -\frac{\cos\theta}{3}[2 + sen^2\theta]$, $\int\cos^3\theta\, d\theta = \frac{sen\theta}{3}[2 + \cos^2\theta]$

- **Escoamento Hidrodinâmicamente Desenvolvido**
- **Número de Reynolds**: $Re = \dfrac{\rho\, u_m D_h}{\mu}$ **Diâmetro Hidráulico:** $D_h = \dfrac{4\, A_t}{P_m}$
- **Potência:** $Pot = F\, V$, $Pot = Q\, \Delta p$,
  $Q = u_m A_T = \int_A V dA$
  - **fator de atrito:** $f = \dfrac{\left[-\partial p / \partial x\right] D_h}{0{,}5 \quad \rho\, u_m^2}$



- Regime laminar $Re = \dfrac{\rho V_m D_h}{\mu} \leq 2300$
- placas paralelas: **fRe**=96, Duto circular: **fRe**=64
- Regime turbulento $Re = \dfrac{\rho\, V_m\, D_h}{\mu} > 2300$
- $\dfrac{1}{f^{\,0,5}} = -2{,}0\, \log\left[\dfrac{\varepsilon / D_h}{3{,}7} + \dfrac{2{,}51}{Re\, f^{\,0,5}}\right]$
- estimativa inicial de Miller $f_o = 0{,}25\left[\log\left(\dfrac{\varepsilon / D_h}{3{,}7} + \dfrac{5{,}74}{Re^{0,9}}\right)\right]^{-2}$



- **Equações da Camada Limite Bi-dimensional**

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \quad \text{e} \quad \rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2} \qquad -\frac{dp}{dx}=\rho U_\infty \frac{dU_\infty}{dx}$$

- **Solução de Blasius:** $Re_L \le Re_c$, $Re_c = 5 \times 10^5$ $\quad Re_x = \dfrac{\rho U_\infty x}{\mu}$

espessura da camada limite: $\dfrac{\delta}{x}=\dfrac{5}{\sqrt{Re_x}}$ **perfil de velocidade aproximado** $\dfrac{u}{U_\infty}=\dfrac{3}{2}\left(\dfrac{y}{\delta}\right)-\dfrac{1}{2}\left(\dfrac{y}{\delta}\right)^3$

fator de atrito: (a) local: $Cf(x)=\dfrac{\tau_s}{0{,}5\,\rho\,U_\infty^2}=\dfrac{0{,}664}{\sqrt{Re_x}}$ **(b) médio:** $\overline{Cf}_L=\dfrac{\overline{\tau_s}}{0{,}5\,\rho\,U_\infty^2}=\dfrac{1{,}328}{\sqrt{Re_L}}$

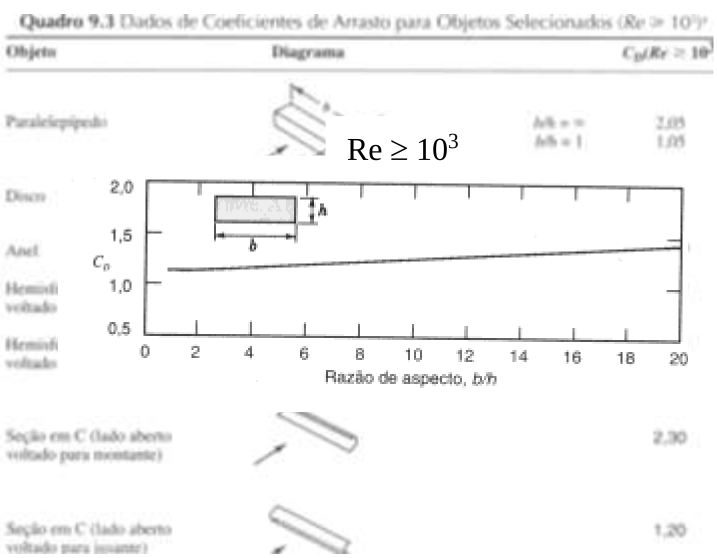
- **escoamento turbulento** $Re_L > Re_c = 5 \times 10^5 \Rightarrow$ turbulento

espessura da camada limite: $\dfrac{\delta}{x}=\dfrac{0{,}381}{\mathbf{Re}_x^{1/5}}-\dfrac{10270}{\mathbf{Re}_x}$, **perfil de velocidade** $\dfrac{u}{U_\infty}=\left(\dfrac{y}{\delta}\right)^{1/7}$

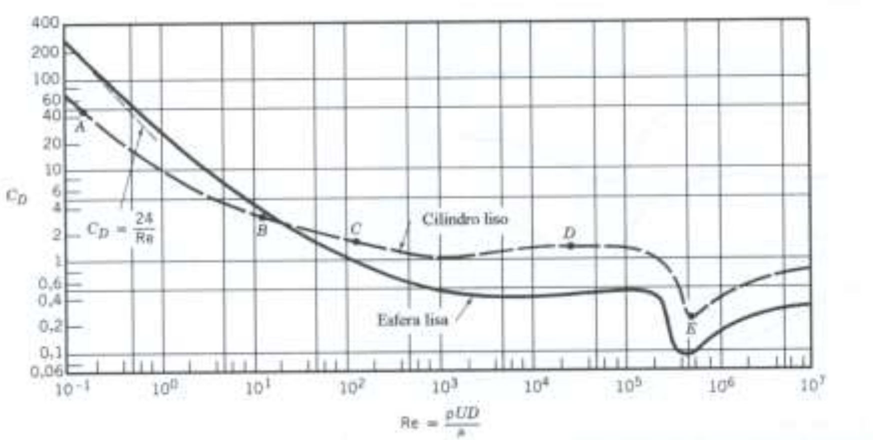
- **fator de atrito: (a) local:** $Cf(x)=\dfrac{\tau_s}{0{,}5\,\rho\,U_\infty^2}=\dfrac{0{,}0592}{Re_x^{1/5}}$ para $R_{ex}<10^7$ **(b) médio:** $\overline{Cf}_L=\dfrac{\overline{\tau_s}}{0{,}5\rho U_\infty^2}$

$\overline{Cf}_L=\dfrac{\overline{\tau_s}}{0{,}5\,\rho\,U_\infty^2}=\dfrac{0{,}074}{Re_L^{1/5}}-\dfrac{1740}{Re_L}$, $Re_L<10^7$ ou $\overline{Cf}_L=\dfrac{0{,}455}{\left(\log Re_L\right)^{2{,}58}}-\dfrac{1610}{Re_L}$, $Re_L<10^9$

- **Coeficiente de Arraste (C_D) e Coef. de Sustentação (C_L):** $C_D=\dfrac{F_D/A_{ef}}{\rho V^2/2}, C_L=\dfrac{F_L/A_{ef}}{\rho V^2/2}$



- Escoamento perpendicular a uma placa



- coeficiente de arraste para esfera e cilindro

FORMULÁRIO – ESCOAMENTO COMPRESSIVEL

- **Ar :** R = 287 Nm/(Kg K), k = cp/cv = 1,4, cp= 1004 N m/(Kg K)
- **gás perfeito:** $p = \rho R T$, $dh = cp\, dT$, $du = cv\, dT$, $h = u + p v$, $v = 1/\rho$,
  $s_2 - s_1 = cp \ln(T_2/T_1) - R \ln(p_2/p_1)$
- **Número de Mach:** M = V/c , $c = \sqrt{k R T}$

- **Equações de conservação Uni-dimensionais**

1) $\rho_1 V_1 A_1 = \rho_2 V_2 A_2 = \dot{m}$

2) $R_x + p_1' A_1 - p_2' A_2 = \dot{m}(V_2 - V_1)$

3) $\dot{Q}/\dot{m} = (h_2 + V_2^2/2) - (h_1 + V_1^2/2)$

4) $\int_{SC} \frac{\dot{Q}/A}{T} dA \le \dot{m}(s_2 - s_1)$, $s_2 - s_1 = cp \ln(T_2/T_1) - R \ln(p_2/p_1)$

5) $\frac{p_1}{\rho_1 T_1} = \frac{p_2}{\rho_2 T_2}$

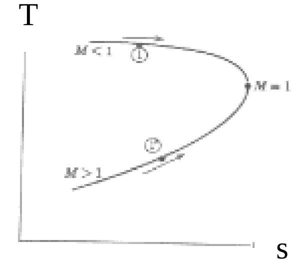
6) $h_2 - h_1 = cp(T_2 - T_1)$

- **Escoamento isoentrópico:** p/ρ^k = constante

$$\frac{T_o}{T} = 1 + \frac{k-1}{2} M^2 \quad , \frac{p_o}{p} = \left(\frac{T_o}{T}\right)^{k/(k-1)} , \quad \frac{\rho_o}{\rho} = \left(\frac{T_o}{T}\right)^{1/(k-1)} , \quad \frac{A}{A^*} = \frac{1}{M}\left[\frac{1 + 0,5\,(k-1)\,M^2}{1 + 0,5\,(k-1)}\right]^{\frac{k+1}{2(k-1)}}$$

- **Choque Normal:** $M_2^2 \left(\frac{\frac{2}{k-1} + M_2^2}{(1 + kM_2^2)^2}\right) = \left(\frac{\frac{2}{k-1} + M_1^2}{(1 + kM_1^2)^2}\right) M_1^2$, $\frac{p_{o2}}{p_{o1}} = \frac{p_{o2}}{p_2}\frac{p_2}{p_1}\frac{p_1}{p_{o1}}$

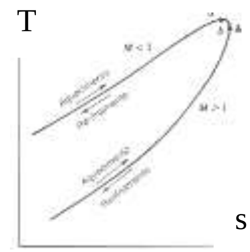
$$\frac{T_2}{T_1} = \frac{1 + 0{,}5\,(k-1)\,M_1^2}{1 + 0{,}5\,(k-1)\,M_2^2} \quad , \quad \frac{p_2}{p_1} = \frac{1 + k\,M_1^2}{1 + k\,M_2^2} \quad , \quad \frac{\rho_2}{\rho_1} = \frac{M_1}{M_2}\sqrt{\frac{T_1}{T_2}}$$



- **Fanno**: $\frac{T}{T^*} = \frac{k+1}{2\left[1 + \frac{k-1}{2} M^2\right]}$; $\frac{\rho}{\rho^*} = \frac{V^*}{V} = \frac{1}{M}\sqrt{\frac{T^*}{T}}$; $\frac{p}{p^*} = \frac{1}{M}\sqrt{\frac{T}{T^*}}$

$$\frac{p_o}{p_o^*} = \frac{1}{M}\left[\left(\frac{2}{k+1}\right)\left(1 + \frac{k-1}{2}\right) M^2\right]^{(k+1)/[2(k-1)]} ;$$

$$\frac{f\,L_{max}}{D_h} = \frac{1 - M^2}{k M^2} + \frac{k+1}{2\,k}\ln\left[\frac{(k+1)\,M^2}{2\left(1 + \frac{k-1}{2} M^2\right)}\right]$$



- **Rayleigh:** $\frac{p}{p^*} = \frac{1+k}{1+k\,M^2}$ $\frac{T}{T^*} = M^2\left(\frac{p}{p^*}\right)^2$; $\frac{\rho^*}{\rho} = \frac{V}{V^*} = M\sqrt{\frac{T}{T^*}}$

$$\frac{p_o}{p_o^*} = \left[\frac{\left(1 + \frac{k-1}{2} M^2\right)}{\frac{k+1}{2}}\right]^{\frac{k}{k-1}} \left[\frac{k+1}{1+k\,M^2}\right] \quad ; \quad \frac{T_o}{T_o^*} = \frac{2\left[1 + \frac{k-1}{2} M^2\right]}{k+1}\left[\frac{M\,(k+1)}{1+k\,M^2}\right]^2$$